

BI-TP 94/08
February 1994

THE NONABELIAN SCREENING POTENTIAL BEYOND THE LEADING ORDER

R. Baier and O. K. Kalashnikov ¹

Fakultät für Physik
Universität Bielefeld
D-33501 Bielefeld, Germany

Abstract

The nonabelian screening potential is calculated in the temporal axial gauge. The Slavnov-Taylor identity is used to construct the three-gluon vertex function from the inverse gluon propagator. After solving the Schwinger - Dyson equation beyond leading order we find that the obtained momentum dependence of the gluon self-energy at high temperature does not correspond to an attractive QCD - Debye potential, but instead it is repulsive and power behaved ($\simeq 1/r^6$) at large distance.

¹Permanent address: Department of Theoretical Physics, P.N.Lebedev Physical Institute, Russian Academy of Sciences, 117924 Moscow, Russia. E-mail address: kalash@td.fian.free.net

1 Introduction

In recent papers [1,2] Rebhan has investigated the screening (Debye) mass for QCD at finite temperature beyond the leading order, especially under the aspect of its gauge dependence (in the case of covariant gauges). This analysis has been performed in the framework of the Braaten - Pisarski resummation scheme [3], and it is successful to define a gauge-independent screening length in next-to-leading order. However, it is logarithmically sensitive to the momentum scale $O(g^2T)$, e.g. to the nonperturbative gluon magnetic mass, and it differs from previously found results [4,5].

On the other hand possible modifications of Debye screening have been discussed in Refs.[6,7], where it is shown that the momentum dependence of the gluon self-energy may essentially affect the large distance behaviour of the potential.

In this paper we calculate the static QCD colour-singlet potential beyond the leading order by exploiting the infrared limit of the (time-time component of the) gluon self-energy $\Pi_{44}(0, \mathbf{k})$. We work in the temporal axial gauge, and we use the full Schwinger-Dyson equation for Π_{44} , which includes the ring graphs as well as nonperturbative vertex corrections. Gauge covariance is guaranteed by the exact Slavnov-Taylor identities in order to construct the three-gluon vertex [8,9].

As a result we find strong indications for a power-like screening potential for high temperature QCD.

2 The gluon polarization tensor and the three-gluon vertex

The following calculations are performed in the temporal axial gauge [4], which is well suited to keep gauge covariance and to evaluate the necessary infrared limit. In this gauge, in which the gauge vector is parallel to the velocity vector u_μ of the medium, the thermal Green function technique is considerably simplified, especially since the exact gluon propagator has only non-vanishing spatial components,

$$\mathcal{D}_{ij}(k) = \frac{1}{k^2 + G} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) + \frac{1}{k^2 + F} \frac{k^2}{k_4^2} \frac{k_i k_j}{\mathbf{k}^2}, \quad (1)$$

where we suppress the colour indices. \mathcal{D}_{ij} only depends on two scalar functions $F(k)$ and $G(k)$, which are related to the gluon polarization tensor $\Pi_{\mu\nu}$

as follows

$$G(k) = \frac{1}{2} \left(\sum_i \Pi_{ii} + \frac{k_4^2}{\mathbf{k}^2} \Pi_{44} \right), \quad F(k) = \frac{k^2}{\mathbf{k}^2} \Pi_{44}. \quad (2)$$

A further important advantage of the axial gauge is provided by the exact Slavnov-Taylor identity for the three-gluon vertex function in the form

$$r_\mu \Gamma_{\mu\nu\gamma}^{abc}(r, p, q) = ig f^{abc} [\mathcal{D}_{\nu\gamma}^{-1}(p) - \mathcal{D}_{\nu\gamma}^{-1}(q)], \quad (3)$$

where f^{abc} denote the structure constants of the colour group.

This equation being rather simple is used to construct the nonperturbative vertex function necessary for calculating the gluon-self energy in the infrared limit, namely the $\Pi_{44}(0, |\mathbf{q}|)$ function, which is required to determine the static screening potential.

The exact nonperturbative graph representation for this tensor is well-known (see e.g. Refs.[4,9]). In the axial gauge it consists of four diagrams for the pure gluon contribution. However, when one considers the Π_{44} -component only two nonperturbative diagrams contribute (the more complicated ones are vanishing). At finite temperature $T = 1/\beta$ the analytical expression for these two nonvanishing graphs is found to be

$$\begin{aligned} \Pi_{44}(0, |\mathbf{q}|) &= \frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \mathcal{D}_{ii}(p) \\ &\quad - \frac{g^2 N}{2\beta} \sum_{p_4} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (2p_4) [\mathcal{D}_{li}(p+q) \Gamma_{ij4}(p+q, -p, -q) \mathcal{D}_{jl}(p)], \end{aligned} \quad (4)$$

where $\Gamma_{ij4}^{abc} = -ig f^{abc} \Gamma_{ij4}$. All functions \mathcal{D}_{ij} and Γ_{ij4} are understood to be exact ones. The main problem is to find the nonperturbative expression for the $\Gamma_{ij4}(r, p, q)$ -function.

Since we do not know the exact expression for the three-gluon vertex function, we try a nonperturbative ansatz, which we exploit in the following. First the vertex function is decomposed as

$$\Gamma_{4ij}^{abc}(q, r, p) = \Gamma_{4ij}^{abc}(q, r, p)^L + \Gamma_{4ij}^{abc}(q, r, p)^T, \quad (5)$$

and its transverse and longitudinal parts are constructed independently. In the following the longitudinal one is used in the form, which has been found

in Ref.[8]:

$$\begin{aligned}
\Gamma_{4ij}^{abc}(q, r, p)^L = & -igf^{abc} \left\{ \delta_{ij}(r_4 - p_4) - \frac{1}{r^2 - p^2} \left[\left(\frac{G(r)}{\mathbf{r}^2} - \frac{F(r)}{r^2} \frac{r_4^2}{\mathbf{r}^2} \right) \right. \right. \\
& - \left. \left(\frac{G(p)}{\mathbf{p}^2} - \frac{F(p)}{p^2} \frac{p_4^2}{\mathbf{p}^2} \right) \right] [(\mathbf{p}\mathbf{r})\delta_{ij} - p_i r_j](r_4 - p_4) \\
& + \delta_{ij} \left(r_4 \frac{F(r)}{r^2} - \frac{F(p)}{p^2} p_4 \right) + \frac{1}{q^2 - r^2} \left(\frac{F(q)}{q^2} - \frac{F(r)}{r^2} \right) q_i r_4 (q - r)_j \\
& + \frac{1}{p^2 - q^2} \left(\frac{F(p)}{p^2} - \frac{F(q)}{q^2} \right) (p - q)_i p_4 q_j \\
& \left. - \frac{1}{r^2 - p^2} \left(\frac{F(r)}{r^2} - \frac{F(p)}{p^2} \right) r_4 p_4 (r - p)_4 \delta_{ij} \right\}, \tag{6}
\end{aligned}$$

and which is assumed to be valid for any set of momenta including the soft ones. Eq.(6) satisfies the exact Slavnov-Taylor identity by using the standard inverse gluon propagator

$$\mathcal{D}_{ij}^{-1}(p) = \left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2} \right) (p^2 + G(p)) + \left(1 + \frac{F(p)}{p^2} \right) p_4^2 \frac{p_i p_j}{\mathbf{p}^2}, \tag{7}$$

and more details of its properties can be found in Refs.[9,10].

The transverse part of the $\Gamma_{4ij}^{abc}(q, r, p)$ - function is not constrained by the Slavnov-Taylor identity of Eq.(3), but it is introduced to cancel the leading infrared singularities of the longitudinal vertex function (6). It is given by

$$\begin{aligned}
\Gamma_{4ij}^{abc}(q, r, p)^T = & -igf^{abc} \left\{ \frac{-(\mathbf{p}\mathbf{q})\delta_{ij} + p_i q_j}{q^2 - r^2} \left(\frac{F(q)}{q^2} - \frac{F(r)}{r^2} \right) r_4 \right. \\
& \left. - \frac{-(\mathbf{r}\mathbf{q})\delta_{ij} + r_j q_i}{q^2 - p^2} \left(\frac{F(q)}{q^2} - \frac{F(p)}{p^2} \right) p_4 \right\}, \tag{8}
\end{aligned}$$

and together with the transverse part of the function $\Gamma_{44j}^{abc}(q, r, p)$

$$\Gamma_{44j}^{abc}(q, r, p)^T = -igf^{abc} \left\{ \frac{-(\mathbf{p}\mathbf{r})q_j + (\mathbf{p}\mathbf{q})r_j}{q^2 - r^2} \left(\frac{F(q)}{q^2} - \frac{F(r)}{r^2} \right) \right\}, \tag{9}$$

Eq.(8) is constrained by two identities

$$\Gamma_{4\mu j}^T(q, r, p)r_\mu = 0, \quad \Gamma_{4i\mu}^T(q, r, p)p_\mu = 0. \tag{10}$$

In constructing the vertex function Γ_{4ij} it is necessary to impose that it is well-behaved in the infrared limit, i.e. for $q_4 = 0, \mathbf{q} \rightarrow 0$ (and similar for the momenta p and r). Indeed, it is the case with respect to p and r for the longitudinal and transverse parts separately, but not for the infrared limit in q . However, the sum in Eq.(5) is infrared stable, as one can easily verify. Therefore imposing this condition not only motivates the inclusion of the transverse part Eq.(8), but it also fixes its magnitude and (minimal) structure.

In order to determine $\Pi_{44}(0, |\mathbf{q}|)$ both the longitudinal (6) and transverse (8) vertex functions are taken into account in Eq.(4). In the following we restrict the calculation of Π_{44} only up to the $O(g^3)$ term. In this approximation essential simplifications occur due to the different infrared behaviour of the functions $G(p)$ and $F(p)$ in the static limit. If required the function $G(p)$ may be used as an infrared cutoff (and e.g. be identified with the magnetic mass), but in the final stage of the calculation we set $G = 0$. The dependence on the function $F(p)$ is evaluated in the infrared limit by keeping only the $p_4 = 0$ mode in the Matsubara sum of Eq.(4) after identifying the leading term of the Π_{44} -function by $\Pi_{44} = m^2 = g^2 NT^2/3$. The poles at $p_4 = 0$ are regularized in the standard manner when using the temporal axial gauge by the prescription: $\sum_{p_4} 1/p_4^2 = 0$.

After the lengthy algebra is performed we obtain

$$\begin{aligned} \Pi_{44}(0, |\mathbf{q}|) = & \frac{g^2 N}{3\beta^2} + \frac{2g^2 N}{\beta} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left\{ \frac{m^2 [\mathbf{p}^2 + (\mathbf{p}\mathbf{q})]^2}{\mathbf{p}^2 (\mathbf{p} + \mathbf{q})^2 [\mathbf{p}^2 + m^2] [(\mathbf{p} + \mathbf{q})^2 + m^2]} \right. \\ & \left. - \left(1 - \frac{(\mathbf{p}\mathbf{q})^2}{\mathbf{q}^2 \mathbf{p}^2} \right) \frac{2\mathbf{q}^2}{\mathbf{p}^2 [(\mathbf{p} + \mathbf{q})^2 + m^2]} \right\}, \end{aligned} \quad (11)$$

where all integrals have no divergencies and can be explicitly evaluated.

The final result to order g^3 becomes

$$\begin{aligned} \Pi_{44}(0, |\mathbf{q}|) = & m^2 + \frac{g^2 N m T}{2\pi} \left\{ -\frac{1}{2} + \frac{\pi}{8} \frac{\mathbf{q}^4}{m^3 \sqrt{\mathbf{q}^2}} \right. \\ & \left. + \left[\frac{m^2 - \mathbf{q}^2}{m \sqrt{\mathbf{q}^2}} - \frac{(\mathbf{q}^2 + m^2)^2}{2m^3 \sqrt{\mathbf{q}^2}} \right] \arctan\left[\frac{\sqrt{\mathbf{q}^2}}{m}\right] + \frac{(\mathbf{q}^2 + 2m^2)^2}{4m^3 \sqrt{\mathbf{q}^2}} \arctan\left[\frac{\sqrt{\mathbf{q}^2}}{2m}\right] \right\}, \end{aligned} \quad (12)$$

valid for real values of \mathbf{q} . It is important to note that in the low momentum expansion of $\Pi_{44}(0, |\mathbf{q}|)$ a positive cubic term $|\mathbf{q}|^3$ is present in Eq.(12) -

instead of a negative linear one in the case of the one-loop approximation [11]. It originates from the product of the longitudinal - longitudinal modes of \mathcal{D}_{ij} in Eq.(4).

At $\mathbf{q} = 0$ the static self-energy is

$$\begin{aligned}\Pi_{44}(0) &= m^2 + \frac{g^2 N m T}{4\pi} \\ &= \left[\frac{g^2 N}{3} + \frac{3}{4\pi} \left(\frac{g^2 N}{3} \right)^{3/2} \right] T^2,\end{aligned}\tag{13}$$

as obtained recently in Ref.[12].

For reference we quote here separately the $O(g^3)$ contribution due to the product of the longitudinal and transverse modes of \mathcal{D}_{ij} ,

$$\Pi_{44}^{LT}(0, |\mathbf{q}|) = m^2 + \frac{g^2 N m T}{2\pi} \left[\frac{m^2 - \mathbf{q}^2}{m\sqrt{\mathbf{q}^2}} \arctan\left[\frac{\sqrt{\mathbf{q}^2}}{m}\right] - 1 \right], \tag{14}$$

which coincides with the expression (in the covariant Landau gauge) derived by Rebhan [1,2] in the framework of the Braaten - Pisarski resummation scheme [3].

3 The screening potential beyond the leading term

Applying linear response theory it is possible to relate $\Pi_{44}(0, |\mathbf{q}|)$ to the gluon-electric potential. For the colour-singlet case it is represented by the Fourier transform [4]:

$$\begin{aligned}V(\mathbf{r}) &= -\frac{N^2 - 1}{2N} g^2(T) \int \frac{d^3 q}{(2\pi)^3} \frac{e^{i\mathbf{q}\mathbf{r}}}{\mathbf{q}^2 + \Pi_{44}(0, |\mathbf{q}|)} \\ &= -\frac{N^2 - 1}{2N} \frac{g^2(T)}{2\pi^2 r} \int_0^\infty \frac{dq q \sin(qr)}{q^2 + \Pi_{44}(0, q)},\end{aligned}\tag{15}$$

where $r = |\mathbf{r}|$ and $q = |\mathbf{q}|$.

Considering large distances r it is usually assumed that only the static $\Pi_{44}(0, |\mathbf{q}| = 0)$ -limit is essential for evaluating Eq.(15), and that in this limit the observable screening length is related to the electric mass $m_E^2 = \Pi_{44}(0)$, as defined in the temporal axial gauge. Since the leading term for $g \rightarrow 0$ of this quantity (named as the Debye mass) was found to be gauge-independent

it is the belief that the exponential Debye screening is also the result of multi-loop calculations, although the momentum dependence of the gluon self-energy is expected to have a rather complicated analytical structure, e.g. branch points. Even for the case that the only arising singularities are poles, it cannot be excluded that they are becoming complex ones as solutions of

$$\mathbf{q}^2 + \Pi_{44}(0, \sqrt{\mathbf{q}^2}) = 0, \quad (16)$$

such that oscillations in the r dependence of the potential are resulting.

It was demonstrated in Refs.[6,7] that odd powers of $\sqrt{\mathbf{q}^2}$, with the branch point at $\mathbf{q}^2 = 0$, are responsible for the asymptotic power behaviour of $V(r)$ for large distance r and for high temperature.

In our case the situation is analogous to the one discussed in the paper [6], because of the term in Π_{44} of Eq.(12), which is odd (cubic) in powers of $\sqrt{\mathbf{q}^2}$. In order to facilitate the comparison with the one-loop result the same notation as in Ref.[6] is used in the following. The convenient variables are $q = mz$, $x = mr$ and $t = g^2 NT/(8m) = 3m/(8T)$, and a dimensionless screening function $S(x, t)$ is defined by

$$V(\mathbf{r}) = -\frac{N^2 - 1}{2N} \frac{g^2(T)}{4\pi r} S(x, t), \quad (17)$$

and

$$S(x, t) = \frac{2}{\pi} \int_0^\infty \frac{z dz \sin(xz)}{[z^2 + 1 + f(z) + tz^3/2]}, \quad (18)$$

where the function $f(z)$ is obtained from Eq.(12),

$$f(z) = \frac{2t}{\pi} [-1 + (1 - 4z^2 - z^4) \frac{\arctan(z)}{z} + (z^2 + 2)^2 \frac{\arctan(z/2)}{2z}]. \quad (19)$$

Applying contour integration (with the contour surrounding the first quadrant in the complex z - plane) the leading term in the high (but finite) temperature limit becomes

$$S(x, t) \simeq \frac{t}{\pi} \int_0^\infty \frac{dy y^4 e^{-xy}}{[(1 - y^2 + f(y))^2 + t^2 y^6/4]}, \quad (20)$$

with $f(y) = f(-iz)$ on the first Riemann sheet. Contributions arising from possible (complex) poles and from cuts with imaginary branch points may

be neglected, since they vanish exponentially. The integral (20) is evaluated asymptotically by keeping only the leading term for $y \simeq 0$ in the denominator, since the other contributions become exponentially small for large x , or are subleading for $g \rightarrow 0$. Thus a power-behaved repulsive potential

$$V(r)|_{mr \rightarrow \infty} \simeq \frac{N^2 - 1}{8\pi^2} \frac{3g^2(T)T}{(rm)^6} \quad (21)$$

results.

The dependence of the exact $S(x, t)$ of Eq.(18) on $x = mr$ for a fixed value of t , i.e. of the coupling g respectively, is plotted in Fig. 1 for $t = 0.5$ (Fig. 1a) and for $t = 1.0$ (Fig. 1b). The dotted curve represents the exponential Debye form for $t = 0$ at leading order, $S(x, 0) = \exp(-x)$. The full curve is the result of our analysis with $f(z)$ of Eq.(19) and the dashed curve is from Eq.(14).

One can see that after including the momentum dependence in Π_{44} as described above the exponential behaviour of $S(x, t)$ is significantly modified: screening is increased for values of $x \leq 3$ (that means a positive correction to the Debye mass), but at larger distances there is antiscreening due to the cubic momentum term in Π_{44} with $S(x, t) \simeq -24t/(\pi x^5)$. A similar behaviour has been discussed in Ref.[6] in the case of the one-loop approximation (for a comparison one may see Fig. 1 of Ref.[6]).

The dashed curve in Fig. 1 corresponds to the case given by Eq.(14), which is analogous to the situation discussed in Refs.[1,2] (for the gauge parameter fixed by $\alpha = 0$), but without introducing a magnetic term. No odd terms in the momentum q are present. The screening function $S(x, t)$ tends to become oscillatory around its zero value²; this behaviour is more transparent for increasing values of the coupling as illustrated in Fig. 1b: for this purpose a rather extreme value of $t = 1.0$ is chosen. For this large value even the full curve shows two oscillations, but for $x > 7.0$ it remains negative and power-behaved. The dashed one shows oscillations for all $x > 10.0$, but which are damped by $\exp(-x)$.

In order to test the reliability of our calculation with respect to its infrared sensitivity we introduce a small magnetic mass term in the transverse part

² But it has been pointed out by A. K. Rebhan that after introducing a small magnetic mass term in Eq.(14) as described in [1,2] the corresponding function $S(x, t)$ does not change its sign, i.e. it remains positive.

of the gluon propagator by assuming a non-vanishing, but constant value for the function G of Eq.(2). Numerically we find that the full curve in Fig. 1 remains stable with respect to this modification. Indeed this should be the case, since the cubic term in the momentum dependence of Π_{44} does not depend on the transverse part of the gluon propagator, i.e. on the magnetic mass.

4 Conclusion

To summarize, we have analysed the static colour-singlet potential $V(r)$ in QCD at high temperature. In the temporal axial gauge and beyond leading order $V(r)$ turns out to be power-behaved ($\simeq 1/r^6$) and repulsive (Eq.(21)). Although the detailed shape of this potential is gauge dependent, the described qualitative features may actually be independent of the chosen gauge, i.e. as a result of the next-to-leading order treatment, the positive cubic term in q of $\Pi_{44}(0, \mathbf{q})$ already reflects the correct behaviour of the gluon self-energy for small momenta $\mathbf{q} \simeq 0$. We expect that this cubic term (contrary to the linear one appearing in the one-loop approximation) remains infrared stable and survives under the further resummations. So the stated problem turns out to be more complicated than for the usually considered ansatz with the Debye mass, and more detailed investigations are desirable as a future task.

Acknowledgements

We are grateful to A. K. Rebhan for useful discussions. R. B. thanks J. Kapusta for pointing out to him the relevance of the temporal axial gauge. O. K. K. would like to thank the colleagues from the Department of Theoretical Physics of the Bielefeld University for the kind hospitality, and acknowledges partial support by "Volkswagen - Stiftung". This research is also supported in part by the EEC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0537 (DG 12 COMA).

References

- 1.) A. K. Rebhan, Phys. Rev. **D48** (1993) R3967.
- 2.) A. K. Rebhan, "Gauge-independent extraction of the next-to-leading-order Debye mass from the gluon propagator", contribution at the *3rd Workshop on Thermal Field Theories and their Applications*, Banff, Canada, August 1993, preprint BI-TP 93/52.
- 3.) E. Braaten and R. D. Pisarski, Phys. Rev. Lett. **64** (1990) 1338; Nucl. Phys. **B337** (1990) 569.
- 4.) K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160** (1985) 477.
- 5.) J. I. Kapusta, *Finite Temperature Field Theory*, Cambridge University Press, Cambridge, England, 1989.
- 6.) C. Gale and J. Kapusta, Phys. Lett. **B198** (1987) 89.
- 7.) J. Kapusta and T. Toimela. Phys. Rev. **D37** (1988) 3731.
- 8.) O. K. Kalashnikov, JETP Lett. **39** (1984) 405.
- 9.) O. K. Kalashnikov, in *Quantum Field Theory and Quantum Statistics* (I. A. Batalin et al., eds.), Adam Hilger, Bristol, 1987.
- 10.) O. K. Kalashnikov, Fortschr. Phys. **32** (1984) 525.
- 11.) O. K. Kalashnikov and V. V. Klimov, Sov. J. Nucl. Phys. **33** (1981) 443; H. A. Weldon, Phys. Rev. **D26** (1982) 1394.
- 12.) O. K. Kalashnikov, "The nonperturbative equation for the infrared $\Pi_{44}(0)$ -limit in the temporal axial gauge", preprint BI-TP 93/77 (December 1993).

Figure Caption

Fig. 1 The screening function $S(x, t)$ as a function of x for two values of $t = 0.5$ (Fig. 1a) and $t = 1.0$ (Fig. 1b) as described in the text. The change in scales (logarithmic and linear) has to be noted.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9402361v1>